The Chance of the Jackpot being Opened at Poker

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SUMMARY

In poker all possible five-card hands are ranked in an order of precedence. The Jackpot is considered opened if at least one player is dealt a pair of Knaves or any higher ranking hand. Required the probability that the Jackpot is opened. This problem is too complicated for a theoretical solution, so an empirical one was obtained by using a KDF 9 computer. The program simulated the dealing of a number of hands to n players $(1 \leqslant n \leqslant 5)$ and those deals in which the Jackpot would have been opened were counted.

THE game of poker is played with the normal pack of 52 cards, by a number of players, usually from two to seven. A poker-hand consists of five cards. All possible five-card hands are considered as arranged in an order of precedence comprising the following classes (here given in descending order): (1) *Straight flush*—five cards of the same suit in sequence; (2) *Fours*—four cards of the same rank (e.g. four Aces); one inessential card; (3) *Full house*—three cards of one rank, two of another; (4) *Flush*—five cards of the same suit; (5) *Straight*—five cards in sequence; (6) *Threes*—three cards of the same rank; two inessential cards; (7) *Two pairs*—two cards of one rank, two of another; one inessential card; (8) *Pair*—two cards of the same rank; three inessential cards; (9) *Miscellaneous*—all hands not included in the foregoing classes. The order of precedence within a class is determined by the rank of the essential and, if necessary, of the inessential cards. Thus, of two flushes or two miscellaneous hands, that with the superior highest card is the better. Equal hands are possible; thus two hands each consisting of two Threes, two Fours and a Five. In sequences, the Ace can rank either high or low ("round-the-corner" sequences, such as Queen, King, Ace, Two, Three, are, however, not accepted); otherwise it ranks high.

There is a form of the game called a Jackpot. Here it will be sufficient to say that a hand is dealt to each player and the Jackpot may be considered "opened" if at least one player has a pair of Knaves or any hand that ranks higher than this in the order of precedence.

We require the probability that, with n players, the Jackpot is opened. Clearly, this problem is too complicated for a theoretical solution. A computer was therefore used to obtain an empirical one.

The program simulated the dealing of hands to n players $(1 \leqslant n \leqslant 5)$ by the following method. At the start of each deal a list of the numbers 1–52 was set up. A pseudo-random number† in the range 1–52 was calculated and the corresponding entry in the list removed to be the first card dealt. The list was now of length 51 and another

† The sequence of pseudo-random numbers was formed by using a congruence of the type $x_{n+1} \equiv ax_n + b \bmod (c)$. This sequence was rectangularly distributed in the interval (0, 1) and the elements were scaled by an appropriate factor according to the current length of the list. Three different sequences, determined by the initial values $x_0 = 3, 5, 7$, were used and there was no significant difference between the three sets of results.

pseudo-random number in the range 1–51 was used to choose the second card. This was repeated until five cards had been drawn and the resulting hand was then evaluated. This process was repeated for each of the n players in turn and the highest valued hand recorded in the summary after which the original state of the list (1–52) was restored.

To evaluate a hand needs basically a number of tests but in order to speed up the process some short cuts were used.

Let $V1$, $V2$, $V3$, $V4$, $V5$ be the "values" of the cards in the range 1–52 ($2C \equiv 1$, $AS = 52$); let $R1, \ldots$ be the ranks and $S1, \ldots$ the suits.

1. If $S1 = S2 = S3 = S4 = S5$ we have a flush (and must also test for a straight flush).
2. If there is no flush, we count how many cards of each possible rank (1–13) are present.

Ex.	*Hand*	*Ranks*	1	2	3	4	5	6	7	8	9	10	11	12	13	S^2
	3467Q				1	1		1	1					1		5
	33459				2	1	1				1					7
	33359				3		1				1					11
	333AA				3										2	13

The sum of the numbers in each row is always 5 but the sums of their squares are different and in fact determine the type except for straights (or flushes). Further, the value within the type can be easily picked out.

3. If we consider the straight, 45678, the sum of the ranks is 30 and the sum of their squares is 190. Although other hands may also have a sum of 30, the sum of squares will be different. This was the basis of the test for straights.

The program was written in the I.C.I. Compatible-Autocode and run on an English Electric KDF9: the speed of simulation averaged 7,000 hands per minute.

TABLE 1

Hand	*Our results (No. out of 100,000)*	*Kempson's chance (1 in —)*	*Our chance (1 in —)*
Straight flush	0	64,974	—
Fours	28	4,165	3,571·4
Full house	128	694	781·3
Flush	206	509	485·4
Straight	426	255	234·7
Threes	2,117	47	47·2
Two pairs	4,718	21	21·2
Pair	42,170	2½	2·4
Miscellaneous	50,206	2	2·0

We consider first the results of 100,000 deals to one player; these were made as a check. The number of ways of dealing the different poker-hands is, of course, well known; Kempson (1952, p. 89) gives the "chances of being dealt poker combinations in the original five cards" (thus, for "straight flush", he gives "1 in 64,974"). Table 1 gives, respectively, our results, Kempson's chance and the chance derived from our results expressed in the same way as his (to the nearest tenth). It will be seen that our results show very good agreement with those to be expected in actual poker. A χ^2-test confirms that this is indeed so.

Table 2 gives the results of 50,000 deals with each of the three pseudo-random numbers 3, 5, 7, to each of four sets of players, the sets consisting of two, three, four

TABLE 2

(Each column is out of 50,000 deals)

	Pseudo-random number generator					
	3	*5*	*7*	*3*	*5*	*7*
		Two players			*Three players*	
Pairs J	2,641	2,669	2,658	3,254	3,266	3,154
Q	2,803	2,786	2,887	3,444	3,445	3,575
K	2,886	2,795	2,856	3,894	3,787	3,801
A	2,953	3,031	2,980	4,114	4,167	4,053
Two pairs	4,515	4,490	4,546	6,280	6,412	6,386
Threes	2,109	2,062	2,116	2,951	3,013	2,952
Straight	385	358	375	602	551	586
Flush	180	207	215	303	333	280
Full house	143	161	154	201	213	226
Fours	26	19	28	49	39	30
Straight flush	1	1	2	4	2	4
		Four players			*Five players*	
Pairs J	3,449	3,520	3,564	3,511	3,491	3,564
Q	4,005	4,003	3,890	4,089	4,282	4,177
K	4,569	4,460	4,542	4,869	4,970	4,952
A	5,014	5,166	5,130	5,746	5,741	5,814
Two pairs	8,005	7,987	8,029	9,672	9,592	9,353
Threes	4,040	3,947	4,016	4,814	4,833	4,965
Straight	728	758	760	1,015	970	972
Flush	399	355	435	496	494	498
Full house	277	240	305	371	355	342
Fours	53	54	58	60	61	59
Straight flush	2	2	3	4	3	4

and five players, respectively. Thus the entry "6,412" in the "two pairs" row and the "Three–5" column means that, with pseudo-random number 5 and three players, there were, out of 50,000 deals, 6,412 in which one or more of the players had two pairs and none of them had a better hand than this.

The number of times, out of 150,000, in which the Jackpot is opened with two players can now be obtained by adding together all the 33 entries of the two-block (and similarly for three, four and five players). Table 3 (of self-evident form) gives the results.

In poker it is often the custom to make the Jackpot "run" by the following procedure. If the Jackpot is not opened, further stakes are put up, and a "Queenpot" (defined similarly to the Jackpot) is played. Kingpot, Acepot, Two-pairs-pot, Acepot, Kingpot, Queenpot, Jackpot, Queenpot, ... follow until the pot *is* opened. Table 4 is similar to Table 3 and gives the results for a Two-pairs-pot.

TABLE 3

	Number of players			
	2	*3*	*4*	*5*
Number of deals (out of 150,000) in which Jackpot is opened	56,038	75,371	91,765	104,139
Probability of this opening (to three figures)	0·374	0·502	0·612	0·694
Approximate odds of this opening	3 : 2 against	Evens	3 : 2 on	7 : 3 on

TABLE 4

	Number of players			
	2	*3*	*4*	*5*
Number of deals (out of 150,000) in which Two-pairs-pot is opened	22,093	31,417	40,453	48,933
Probability of this opening (to three figures)	0·147	0·209	0·270	0·326
Approximate odds against this opening	7	5	3½	3

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REFERENCE

KEMPSON, E. (1952). *Hoyle up-to-date. The Official Rules of Card Games* (ed. E. Kempson *et al.*). London: Derricke Ridgeway.